Abstract

In this work a new idea is proposed to analyze rough a surface, which supplements the surface texture standard of ISO. The method is based on wavelet multi-resolution decomposition and allows us to give mathematical representation of a rough surface of complex geometry. The proposed method recognizes a rough surface as one body of signal as a whole, contrary to the idea of ISO who represents a surface by indentation profiles along two normal lines. Surface indentation is resolved into multi-wave length bands as compared with two-bands decomposition of ISO. The proposed method gives more detailed description of a rough surface than ISO.

1 Introduction

Surface roughness modifies largely turbulence structure of a flow over the surface and accordingly, its friction drag and heat transfer. So, the effect of roughness must be incorporated in the analysis of the flow, as precisely as possible. However, size of roughness elements is usually far smaller than global scale of a flow field and varies locally both in height and in plane. In order to avoid prohibitively huge memory size to store geometry of rough surface indentation faithfully, modeling of the surface is inevitable. For mean flow analysis, mean roughness height $y_s$ in wall unit is used for sand-grain roughness and equivalent sand-grain roughness height is defined also for regularly distributed roughness elements of canonical shapes. This simple concept has been confirmed to be useful for flows in certain practical flows such as flow in a long pipe and flow over a wall of periodic roughness pattern. Rough walls in practical cases are usually in a limited area and not in whole surface or non-uniform such that smooth wall suddenly transfers to rough surface. These non-uniform rough walls are likely to be represented better by a smooth wall on which roughness elements of different size and random separation are mounted. The method named discrete element model is based on this concept and its formulation for Reynolds averaged flow was established and was confirmed to be valid for pipe flows [1][2]. Two extreme cases of roughness elements are equivalent isolated elements of no mutual interference and continuous elements like sand-grain roughness, and normal walls are in between. Series of this work intends to propose a new method to incorporate rough surface in a flow analysis, by a simplified model. Of wide scope of the study, present report describes an important part of extracting substantially important peaks from non-uniform rough surface.

Framework of the present idea is illustrated in Fig.1 and is as follows. Roughness height is not uniform usually and tall elements are expected to be most effective in modifying a flow. Accordingly, tall and influential peaks should be picked up by some adequate means. Then, picked-up peaks are regarded as representing the whole roughness, low peaks being neglected. The drag of picked-up peaks is estimated by substituting that of equivalent hemisphere. Thereafter, the drag is implemented
in flow analysis so that same amount of drag is computed as friction drag in a circle on the wall. Thus, a rough wall can be treated as a plane wall having locally different friction. Present article describes a proposal how to identify geometry and the size of influential peaks, by coarsening the rough surface. Two-dimensional multi-resolution wavelet decomposition is applied for this purpose. In this method, whole surface is treated as a unit body of signal, contrary to the conventional ISO method which combines indentations along two mutually normal straight lines. Each component of the indentation $g^{(j)}(x,y)$ of the decomposition represents surface indentation in $j$-th band-width which is in wave number between $2^{N+j}$ and $2^{N+j+1}$, in both $x$- and $y$-direction, where $j=0$ – $N$ and $N$ is the total number of wave number region, and $j$ is integral number. Primary signal including wave number from zero to largest wave number( $2^N$ ) is numbered 0 -th signal $f^{(0)}(x,y)$. Decomposition is the operation so that the wave-number components $g^{(j)}(x,y)$ are separated consecutively in the order from the largest wave-number to the next. Inverse application of the operation allows us to design a rough surface, in which one adds wave-number component $g^{(j)}(x,y)$ from lowest one to the next to an original most slowly varying wavy surface. The decomposition is directly applied to extract important peaks, and design is used to generate rough wall of various kind of properties in the future work, to examine the validity of a model.

## 2 Wavelet Multi-Resolution Decomposition

To begin with, one-dimensional wavelet decomposition is briefly described in order to give the principle of the method [3][4]. An arbitrary random signal $f(x)$ is decomposed into $N$ components $g^{(j)}(x)$ of consecutive wave-number region of band-width $2^{N+j}$. For this purpose, scaling function $\phi(x)$ and corresponding mother wavelet $\psi(x)$ are introduced which are defined as follows.

$$\phi(x) = \sum_{k} p_k \phi(2x-k),$$

$$\psi(x) = \sum_{k} q_k \phi(2x-k)$$

(1)

where $k$ is integer and coefficients $p_k$ and $q_k$ are specific for each individual scaling function adopted in each problem. Above two functions satisfy two-scale relationship and $\phi(x)$ is decomposed as

$$\phi(2x-l) = \frac{1}{2} \sum_{k} \{ g_{2k-l} \phi(x-k) + h_{2k-l} \psi(x-k) \}$$

(2)

where both $k$ and $l$ are integral numbers and coefficients $g_k$, $h_k$ are also specific for scaling function. Using these two functions, $j$ -th components of an arbitrary one-dimensional random signal $f^{(j)}(x)$ are defined as:
\[ f^{(j)}(x) = \sum_{k} c_k \phi(2^j x - k), \]
\[ g^{(j)}(x) = \sum_{k} d_k \psi(2^j x - k) \]

These equations compose the identical function \( f^{(j+1)}(x) \) of next level as
\[ f^{(j+1)} = f^{(j)} + g^{(j)} \quad (4). \]

\( f^{(j)}(x) \) is the component of the smaller half wave-number of \( f^{(j+1)}(x) \) and \( g^{(j)}(x) \) is the larger half. Repetition of Eq.(4) gives
\[ f^{(N)} = g^{(N-1)} + g^{(N-2)} + \ldots + g^{(0)} + f^{(0)} \quad (5). \]
Thus, whole signal is decomposed into \( N \) components of different wave-number region.

Analysis is an operation to identify each component \( g^{(j)}(x) \) and design of a random signal is to identify \( f^{(j)}(x) \) by some adequate tool, such as by random number. Analysis determines \( c_l^{(j+1)}(x) \) and \( d_l^{(j+1)}(x) \), knowing \( c_k^{(j)}(x) \), while design determines \( c_k^{(j)}(x) \) by \( c_l^{(j+1)}(x) \) and \( d_l^{(j+1)}(x) \). In both operations, calculations are conducted by simple recurrent procedure.

The principle mentioned above can be extended to two-dimensional case, straightforwardly. Two-dimensional grid system \( k \times l \) in \( x \) - and \( y \) -direction respectively is considered. Scaling function corresponding to Eq.(1) is defined [5][6] as
\[ \Phi_{kl}^{(j)}(x,y)=\phi(2^j x-k)\phi(2^j y-l) \quad (6) \]
where \( k \) and \( l \) are integral numbers. Corresponding mother wavelets are not unique but three and given by
\[ \psi_{1,kl}^{(j)}(x,y)=\psi(2^j x-k)\psi(2^j y-l) \quad , \]
\[ \psi_{2,kl}^{(j)}(x,y)=\psi(2^j x-k)\psi(2^j y-l) \quad , \]
\[ \psi_{3,kl}^{(j)}(x,y)=\psi(2^j x-k)\psi(2^j y-l) \quad (7) \]
Similarly to Eq.(3), \( (j+1) \)-th component of a two-dimensional signal \( f(x,y) \) is decomposed into \( f^{(j)}(x,y) \) and \( g^{(j)}(x,y) \), which are defined as follows.

\[ f^{(j)}(x,y) = \sum_{k} \sum_{l} c_{kl}^{(j)} \Phi_{kl}^{(j)}(x,y) = \sum_{k} \sum_{l} c_{kl}^{(j)} \phi(2^j x-k)\phi(2^j y-l) \quad (8) \]
and
\[ g^{(j+1)}(x,y) = g_1^{(j+1)}(x,y) + g_2^{(j+1)}(x,y) + g_3^{(j+1)}(x,y) \quad (9) \]
where
\[ g_1^{(j)}(x,y) = f_1^{(j)}(x,y) + \sum_{k} \sum_{l} d_{kl}^{(j)} \phi(2^j x-k)\psi(2^j y-l) \]
\[ g_2^{(j)}(x,y) = f_2^{(j)}(x,y) + \sum_{k} \sum_{l} d_{kl}^{(j)} \psi(2^j x-k)\phi(2^j y-l) \]
\[ g_3^{(j)}(x,y) = f_3^{(j)}(x,y) + \sum_{k} \sum_{l} d_{kl}^{(j)} \psi(2^j x-k)\psi(2^j y-l) \quad (10) \]

That is, wavelets are composed of three component wavelets. The band areas of each

(a) Wave-number area \( \omega_x \times \omega_y \) of \( f^{(j)} \).

(b) Wave-number square of \( f^{(j)} \) and \( g^{(j)} \).

Fig.2 Wave number area of each component.
above-mentioned component are illustrated in Fig.2. Figure 2(a) shows that \( f^{(j+1)} \) has wave number square whose length of one side is twice as large as that of \( f^{(j)} \) and Fig.2(b) shows that \( f^{(j)} \) is lower half wave-number component of \( f^{(j+1)} \) in both \( x \) and \( y \) directions and the rest three, i.e. \( g^{(j),i} = 1, 3 \) are upper half wave-number components both in \( x, y \) \((i = 3)\) and partly in \( x \) \((i = 1)\) or in \( y \) \((i = 2)\).

Procedure of downward shift of level (decomposition) and upward shift (design) is again conducted by using simple and recurrent equation described later must be applied for the computation.

In the present analysis, maximum grid numbers in one direction is \( 2^{J+N} \). That is, height at the points \( M = k(=2^{J+N}) \times l(=2^{J+N}) \) is measured. In ISO, representative statistical quantities such as mean absolute height \( \delta_{p_a} \), \( \delta_{n_a} \), \( \sigma_{p_y} \), \( \sigma_{n_y} \), \( \sigma_{p_q} \), \( \sigma_{n_q} \), etc. are defined for each component. Corresponding to these mean, we introduce following two-dimensional mean heights, for each level of resolution \( j \).

\[
\delta_{jd}^{(j+2D)} = \frac{1}{M} \sum_{k} \sum_{l} |f_{kl}^{(j)}| \\
\delta_{jd}^{(j+2D)} = \sqrt{\frac{1}{M} \sum_{k} \sum_{l} (f_{kl}^{(j)})^2} \quad (13)
\]

Above average is taken on the largest grid number of \( 2^{J+N} \), despite that the effective grid number is not same but smaller in lower resolution level. So, interpolation by the equation described later must be applied for the computation.

Figure 3 shows a measured rough surface which is a sand paper surface of Height of the surface at 512 points of every \( 2 \mu m \) in both \( x \) - and \( y \) -direction was measured by laser height meter. The picture is the primary surface \( f^{(0)}(x, y) \) and Fig.3(b) is the surface three levels lower \( f^{(-3)}(x, y) \) obtained by successive decompositions. Gray scale shows the scale of height in \( \mu m \). In this decomposition, the scaling function \( \Phi(x, y) \) in Eq.(6) is defined
Analysis and Design of Irregularly Indented Rough Surface by Wavelets

Fig. 3 Measured rough surface of a sand-paper 1200. Primary data are at every 2 µm.

by adopting B-spline function of rank 4 \( N_4(x) \) as one-dimensional scaling function \( \phi \). Two-scale coefficients \( p_k \) and \( q_k \) for \( N_4(x) \) which appear in Eq.(1) are known[4]. In order to get analytical form of \( \rho^{(0)} \) of Fig.3(a), the coefficient \( c_{kl}^{(0)} \) of primary surface is required. For this, following interpolation formula for \( N_4(x) \) is used. That is, any arbitrary two-dimensional function \( \rho^{(0)}(x, y) \) can be converted to a continuous function, if the values \( \rho_0(k, l) \) at integral grid points \((k, l)\) are known, by

\[
\rho_0(x, y) = \sum_{m=1}^{2^j} \sum_{n=1}^{2^j} c^{(0)}_{mn} N_4(x-m) N_4(y-n) \quad (14)
\]

and the coefficient \( c^{(0)}_{kl} \) is given by

\[
c^{(0)}_{mn} = \sum_{m} \sum_{n} \rho(k,l) \beta_{m+2-l} \beta_{n+2-l} \quad (15)
\]

\[
\beta_m = \sqrt{\frac{3}{2}} \sqrt{2}^m, \quad m = -5 \quad 5
\]

where practically accurate enough result is obtained by truncating at \( m = 5 \).

Figure 4 shows an example of detail of decomposition, namely, from \( j = -2 \) to \( j = -3 \). In the figure, three wavelet components of \( f^{(-3)} \) are demonstrated in the same disposition as Fig.2(b), except that left-bottom corner has \( f^{(-2)} \) in place of \( f^{(-3)} \), which is shown in Fig.3(b). Three components \( g^{(-3)}_i \), \( i = 1 \) to 3, compose \( g^{(-3)} \). Grid number on one side of the square is 128 and 64 for \( j = -2 \) and \( j = -3 \), respectively. In \( g^{(-3)}_1 \), the wave number in \( x \)-direction is half of that in \( f^{(-2)} \) and kept unchanged in \( y \)-direction. This is why one finds peaks elongated in \( x \)-direction. Meanwhile, peaks in \( g^{(-3)}_2 \) are elongated in \( y \)-direction, wave number being smaller in \( y \)-direction. \( g^{(-3)}_3 \) has half wave-length of \( f^{(-2)} \) in both directions and the peaks look like crossing 45 degrees to both directions.

Figure 5 shows the profiles of \( f^{(j)} \)'s \( j = 0 \) to -3 along \( x \)-axis. Fast indentation is gradually removed as the decomposition proceeds.

In Fig. 6, probability density of height of each level \( \delta^{(j)} = f^{(j)}(k, l) \) is given. The density is calculated from the height at grid points of each resolution level. Mean height is not subtracted. It is demonstrated that roughness of a sand paper is a particular surface in which height of roughness elements is of Gaussian distribution at every band of wave number. In addition, peaks in smaller wave number are lower than those in larger wave number.

Table 1 shows the details of mean height of the surface of each resolution level. \( \delta^{(j,2D)}_{sa} \) and \( \delta^{(j,2D)}_{qa} \) are as defined in Eq.(12), while \( d^{(j,2D)}_{ia} \)
Fig. 4 Component surfaces of resolution level $j = -3$

Fig. 5 Profiles $f^{(j)}$ of the surface along $x$-axis.

Fig. 6 Probability density distribution of surface indentation of each level $f^{(j)}$. 
Table 1. Mean height of $f^{(j)}$ and mean of coefficients $d_{i}^{(j)}$.

<table>
<thead>
<tr>
<th>$j$</th>
<th>$\delta_{ja}^{(j,2D)}$</th>
<th>$\delta_{jq}^{(j,2D)}$</th>
<th>$d_{1a}^{(j,2D)}$</th>
<th>$d_{2a}^{(j,2D)}$</th>
<th>$d_{3a}^{(j,2D)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.63</td>
<td>7.41</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>5.03</td>
<td>6.55</td>
<td>19.37</td>
<td>17.24</td>
<td>37.58</td>
</tr>
<tr>
<td>-2</td>
<td>4.52</td>
<td>5.85</td>
<td>20.41</td>
<td>19.39</td>
<td>40.04</td>
</tr>
<tr>
<td>-3</td>
<td>4.03</td>
<td>5.22</td>
<td>17.67</td>
<td>18.02</td>
<td>39.67</td>
</tr>
</tbody>
</table>

and $d_{i}^{(j,2D)}$, $(i=1,2,3)$ are mean absolute value and mean rms-value of coefficient $d_{i}^{(j)}$ of $s_{i}^{(j)}$, based on the same definition as Eq.(12). Both $\delta_{ja}^{(j,2D)}$ and $\delta_{jq}^{(j,2D)}$ do not include mean height. Namely, mean height shown in Fig.6 is subtracted from the original profile. The fact that $\delta_{ja}^{(j,2D)}$ and $\delta_{jq}^{(j,2D)}$ slightly decrease with $j$ is consistent with Fig.6. All $d_{1a}^{(j,2D)}$ and $d_{2a}^{(j,2D)}$ are almost independent on $j$, suggesting that spectrum of indentation profile is nearly uniform. However, it is to be noted that $d_{3a}^{(j,2D)}$ and $d_{qa}^{(j,2D)}$, as well as $d_{1q}^{(j,2D)}$ and $d_{2q}^{(j,2D)}$ are nearly equal magnitude, which is suggestive of homogeneity in plane. Meanwhile, magnitudes of $d_{3a}^{(j,2D)}$ and $d_{qa}^{(j,2D)}$ are about two times as large as those of $i=1,2$. This is because the magnitude of $\phi$ is about two times as large as that of $\psi$. Similar reasoning is given to the difference in order of magnitude between $\delta_{ja}^{(j,2D)}$ and $d_{1a}^{(j,2D)}$ $(i=1,2)$, and to rms-value as well.

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are picked up, by specifying $k = -0.3$. As too many areas of small height are found, low peaks having lower than 20% of tallest one are eliminated thereof. Figure 7(a) shows the retained tall peaks. Then, each retained body is substituted by a hemisphere of equal volume. Figure 7(b) is the hemispheres projecting on a flat plate thus obtained. In the figure, hemispheres lower than 1/3 of the tallest one are again discarded. We expect that Fig.7(b) represents the original rough surface. The profile used for this truncation is again problematic but in this example, $f^{(2)}$ is used.

Present proposal intends to estimate drag of roughness elements by that of hemispheres. Adequate estimate of drag of hemispheres distributed in a scattered manner is also a difficult task.

Much novelty and many examinations required before we reach the goal.

4. Concluding Remarks
A new approach to analyze rough walls by multi-resolution decomposition based on wavelets is proposed. It is intended to pick up important peaks from randomly indented surface in order to analyze efficiently flow over the wall. An example of analysis using a sandpaper surface confirmed that the method gives more detailed description of surface texture than ISO. Inverse application of the method is expected to provide a tool to design a rough wall of any kind of statistical property.

References