Numerical modeling of complex turbulent flows

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Abstract

The work deals with the solution of two cases of turbulent incompressible flows: impinging jet flow and round jet in cross-flow. The flows are considered turbulent and statistically steady. The mathematical model is based on Reynolds averaged Navier-Stokes equations with a two-equation turbulence model. The turbulence models are low Reynolds number eddy-viscosity one and an EARSM model. The system of equations is solved by artificial compressibility method. The discretisation uses finite volume method with higher order upwind approximation for convective term and implicit discretisation in time. For both cases results are compared with experimental data. In the case of impinging jet flow, the heat transfer from impingement wall is also solved.

1 Introduction

In this work, we investigate two cases of 3D turbulent flow using the Reynolds averaging approach. The first one is round jet in cross-flow, the second one impinging jet flow. Using sufficiently accurate implicit finite volume method, we study the influence of turbulence modeling on the results. We use one eddy viscosity and explicit algebraic Reynolds stress model for turbulent momentum transfer, and three different turbulent heat flux closures.

2 Mathematical model

The turbulent flow-field in both cases is assumed to be statistically steady. Then the mean flow-field is governed by Reynolds averaged Navier-Stokes (RANS) equations in Cartesian coordinates $x_i$

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_j u_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} (\nu S_{ij} - \tau_{ij}),$$

where $u_i$ is mean velocity vector, $p$ kinematic pressure, $\nu$ laminar kinematic viscosity, and $\tau_{ij}$ Reynolds stress tensor.

We apply two closures for $\tau_{ij}$: the (extended) eddy viscosity one of SST turbulence model and explicit algebraic Reynolds stress model (EARSM) due to Wallin, Hellsten. The Reynolds stress by SST model reads

$$\tau_{ij} = -\nu_T 2S_{ij} + \frac{2}{3} k \delta_{ij},$$

$$\nu_T = \frac{a_1 k}{\max(a_1 \omega, \Omega F_2)}, \quad a_1 = 0.31, \quad (2)$$

where $k$ is turbulent kinetic energy, $\omega$ specific dissipation rate, $\Omega$ magnitude of vorticity, and $F_2$ a switching function given in [6].
EARSM constitutive relation is given in terms of anisotropy tensor $a_{ij}$ [11, 2]

$$
\tau_{ij} = a_{ij} k + \frac{2}{3} k \delta_{ij}, \quad (3)
$$

$$
a_{ij} = \beta_1 \tau S_{ij} + \beta_3 \tau^2 (\Omega_{ik} \Omega_{kj} - \Omega_{ij} \delta_{ij}/3) + \beta_4 \tau^2 (S_{ik} \Omega_{kj} - \Omega_{ik} S_{kj}) + \beta_5 \tau^2 (S_{ik} \Omega_{kl} \Omega_{lj} + \Omega_{ik} \Omega_{kl} S_{lj} - 2 IV \delta_{ij}/3) + \beta_6 \tau^2 (\Omega_{ik} S_{lj} \Omega_{lm} \Omega_{mj} - \Omega_{ij} \Omega_{kl} S_{lm} \Omega_{mj}),
$$

where $\tau$ is turbulent time scale limited from below by Kolmogorov time scale $\sim \sqrt{\nu/\epsilon}$

$$
\tau = \max \left( \frac{k}{\epsilon}, 6 \frac{\nu}{\epsilon} \right), \quad \epsilon = 0.09 k \omega, \quad (4)
$$

$\Omega_{ij}$ is rotation rate tensor

$$
\Omega_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right), \quad (5)
$$

and $II$, $IV$ are invariants formed from $S_{ij}$, $\Omega_{ij}$ given in [2] as well as the coefficients $\beta$.

Both turbulence models use $k$-$\omega$ model equations of similar form

$$
\frac{Dk}{Dt} = P - \beta^* k \omega + \frac{\partial}{\partial x_j} \left[ (\nu + \sigma_k \nu_T) \frac{\partial k}{\partial x_j} \right],
$$

$$
\frac{D\omega}{Dt} = \gamma \frac{\omega}{k} P - \beta \omega^2 + \frac{\partial}{\partial x_j} \left[ (\nu + \sigma_\omega \nu_T) \frac{\partial \omega}{\partial x_j} \right] + \frac{\sigma_d}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}, \quad (6)
$$

where $D \cdot /Dt \equiv \partial \cdot /\partial t + \partial (u_j) /\partial x_j$, and coefficients for SST model are given in [6]. The effective eddy-viscosity in EARSM is given by [2]

$$
\nu_T = C_\mu k \tau, \quad C_\mu = -\frac{1}{2} (\beta_1 + II_3 \beta_6). \quad (7)
$$

For the remaining coefficients see also [2].

3 Numerical method

The system of RANS and turbulence model equations is solved by an artificial compressibility method, which consists of adding a pressure time derivative into the continuity equation:

$$
\frac{1}{\beta^2} \frac{\partial P}{\partial t} + \frac{\partial u_i}{\partial x_i} = 0, \quad (8)
$$

where the parameter $\beta$ has been chosen equal to the maximum velocity in the solution domain. The steady solution of the original system is obtained for $t \to \infty$.

To discretize the equations in space, the cell-centered finite volume method is used. The grid consists of hexahedral cells and is structured and multi-block. An example of handling circular nozzle in the case of jet in cross-flow is shown in Fig. 1.

The convective terms are discretized using 3rd order van Leer upwind interpolation (without a limiter). The viscous terms are approximated using 2nd order central scheme, with cell face derivatives computed on a dual grid of octahedrons constructed over each face of primary grid. Pressure stabilization in the form of pressure Laplacian is added to the continuity equation to prevent pressure-velocity decoupling.

The time integration uses backward Euler (implicit) scheme. The Newton linearized implicit operator contains also convective terms, however has the 1st order upwind stencil. In EARSM, linear part of turbulent diffusion corresponding to eddy viscosity (7) is discretized implicitly, the remaining part explicitly. Only the negative part of source terms in the turbulence model is discretized implicitly. The resulting block 7-diagonal system of algebraic equations is solved using block relaxation method with direct tri-diagonal solver for selected family of grid lines [3]. The CFL numbers range between 30 and 100.

4 Jet in cross flow

This section deals with the simulation of flow-field generated by a round jet injected into a cross-flow of same fluid. The case modeled is given by the available measurement [10], where the speeds were 40.8 m/s and 10.2 m/s in the jet nozzle and in the cross-flow, respectively, with
no effects of compressibility. The turbulence intensity in both inlets was very low (under 0.2%).
The rectangular computational domain approximating aerodynamic tunnel test section is shown in Fig. 2. The dimensions are \((d, A, B, C, H) = (1, 4, 40, 19, 14.8) \times d\), where \(d = 13.3\) mm, and the Reynolds number for jet nozzle diameter and velocity equals 35200 (air). The following boundary conditions were prescribed for velocity components \(u, v, w\), kinetic energy of turbulence \(k\) and specific dissipation rate \(\omega\) (in non-dimensional form with 40.8 m/s and \(d\) as reference velocity and length):

- \(x = -A\) (cross flow inlet):
  - \(u = U_e = 0.25, v = w = 0\)
  - \(k = 6.75 \cdot 10^{-4}\)
  - \(\omega = k/(0.1\nu)\)

- \(z = 0, (x^2 + y^2) \leq 0.5\) (jet nozzle):
  - \(u = v = 0, w = 1 - (\sqrt{x^2 + y^2}/0.5)^{26}\)
  - \(k = 3 \cdot 10^{-2}\)
  - \(\omega = k/(0.5\nu)\)

- \(x = 40\) (outlet):
  Neumann b.c. for \(u, v, w, k, \omega\)

- \(z = 0\) (bottom wall):
  - \(u = v = w = 0\)
  - \(k = 0\)
  - \(\omega = 60\nu/(\beta_1 y_1^3)\)
• side and upper walls:
  - inviscid (slip) wall, i.e. normal component of velocity = 0,
  - Neumann b.c. for:
    - tangential velocity, \( k, \omega \)

The lateral and upper walls were considered inviscid in order to reduce the number of grid points. This simplification is made possible by the fact, that the flow rate is prescribed not in terms of pressure drop in the channel but directly in terms of velocities, thus there is no need to capture friction on walls exactly, and further justified by the fact, that in the measurement measures to eliminate the displacement effect of neglected boundary layers were taken as well (slightly diverging walls).

Figures 3–5 show profiles of horizontal and vertical velocity component in the mid-plane. The measured values [10] are for two inlet turbulence intensities and different runs. It is apparent that in the simulation the jet bends too slowly with respect to measurements. A test computation with slightly different heights of the domain showed no improvement. Thus we could say that this is not due to the inviscid forces but rather due to the inaccurate model for turbulent mixing. The SST model predicts higher vertical velocity than EARSM, which is caused by worse capturing of the secondary vorticities the jet breaks in, see Fig. 6. Moreover, it seems according to measurements that the flow-field contains large scale unsteady vortices and the errors have more fundamental origin in the steady RANS approach.

5 Impinging jet flow

5.1 Isothermal case

Here we consider the solution of RANS equations with SST and EARSM turbulence models in the case of ERCOFTAC’s impinging jet [1] for nozzle-to-wall distance 2 nozzle diameters \( D \) and nozzle Reynolds number 23000. In the previous works [5, 4] it has been found that eddy viscosity models highly over-predict the thickness of wall jet, not to mention the over-prediction of turbulence kinetic energy around impingement. The improvement brings the Bradshaw hypothesis used in the SST model as an eddy viscosity limiter. The SST model however has poor near wall behavior typical for \( k-\omega \) models, apparent in Fig. 7, where the “kink” in wall shear stress at radial position \( r/D \approx 2 \) is missing. A \( k-\epsilon \) model is capable of predicting the kink and when the SST-like eddy viscosity limiter is used, also the wall jet prediction is correct [4]. In the present work we consider the \( k-\omega \) model only and show the influence of constitutive relation. Indeed, the EARSM improves the \( k-\omega \)’s behavior here. The EARSM and SST provide similar results in terms of radial velocity, Fig. 8, the Reynolds shear stress shown in Fig. 10 seems better in the SST model. The EARSM also attenuates the over-prediction of turbulence kinetic energy at impingement typical for eddy viscosity, see Fig. 9, however the measured value is still well lower.

5.2 Heat transfer

In this section we consider the heat transfer from the heated impingement wall. Since the temperature difference of approx. 25°C as well as velocity is small, the energy equation is simplified to a passive scalar transport equation for temperature \( T \):

\[
\frac{\partial T}{\partial t} + \frac{\partial u_i T}{\partial x_i} = \frac{\partial}{\partial x_i} \left[ \nu \frac{\partial T}{\partial x_i} \right] - \frac{\partial}{\partial x_i} u_i^\prime T^\prime, \quad \text{(9)}
\]

where \( \nu \) is laminar Prandtl number and \( u_i^\prime T^\prime \) turbulent heat transfer.

Three closure models for \( u_i^\prime T^\prime \) were used. First one is the eddy diffusivity model

\[
\overline{u_i^T} = -\frac{\nu_T}{P_{RT}} \frac{\partial T}{\partial x_i}, \quad \text{(10)}
\]

where \( \nu_T \) is eddy viscosity and \( P_{RT} \) turbulent Prandtl number set equal to 0.85. Then, with the anisotropy models for Reynolds stress we used two anisotropic models for heat transfer. The generalized gradient-diffusion hypothesis (GGDH) model reads

\[
\overline{u_i^T} = -C_t \overline{u_i u_j^\prime} \frac{\partial T}{\partial x_j}, \quad C_t = 0.3, \quad \text{(11)}
\]
where $\tau$ is the turbulent time scale. In the fully turbulent region, $\tau = k/\epsilon$. The lower limit is given by Kolmogorov time scale $\sqrt{\nu/\epsilon}$, whereas the realizability gives the upper limit. The final expression is [8]

$$
\tau = \min \left[ \max \left( \frac{k}{\epsilon}, 6\sqrt{\nu/\epsilon} \right), \frac{C_R}{6C_\mu T_S} \right],
$$

$C_R = 0.46$, $C_\mu = 0.09$. (12)

The third model considered, which formally includes the previous two, is the one of Abe, Suga (AS) [9].

$$
\begin{align*}
\bar{u}'_i T' &= -c_t \tau (\alpha) \frac{\partial T}{\partial x_j}, \\
\sigma_{ij} &= c_{\sigma 0} \delta_{ij} + c_{\sigma 1} \bar{u}'_i \bar{u}'_j / k + \\
&\quad + c_{\sigma 2} \bar{u}'_i \bar{u}'_j / k^2, \\
\alpha_{ij} &= c_{\alpha 0} \tau \Omega_{ij} + \\
&\quad + c_{\alpha 1} \tau (\Omega_{ij} \bar{u}'_i \bar{u}'_j / k + \Omega_{ij} \bar{u}'_i \bar{u}'_j / k),
\end{align*}
$$

(13)

where the coefficients are

$$
c_t = \frac{0.38}{(1 - \exp(-R_T/100))^{1/4}},
$$

$c_{\sigma 0} = 0$, 
$c_{\sigma 1} = 0.2 f_b + 0.1 f_{Pr}$, 
$c_{\sigma 2} = 1 - f_b - f_{Pr}$, 
$c_{\alpha 0} = 0$, 
$c_{\alpha 1} = (1 - f_{Pr}) \cdot \left[ -0.5 g_A \xi - \frac{0.02 \exp[-(\xi/2.2)^2]}{1 + 5\xi^2 - g_A + (\xi + 0.2)^2} \right],
$$

$f_b = (1 - f_{Pr})^2 \exp[-(\xi/2.2)^2 - (g_A/0.3)^2],
$$

$f_{Pr} = (1 + (Pr/0.85)^{1.5})^{-1}$,

$g_A = 0.3(1 - \exp[-(R_T/70)^2])$, 

$$
\xi = \tau \sqrt{2S_i S_{ij}},
$$

(14)

and the turbulent time scale $\tau$ is taken same as in the EARSM, Eq. (4).

### 5.3 Heat transfer results

The temperature of impingement wall is kept constant at $T_w = 313$ K and the temperature of fluid at nozzle exit was $T_0 = 293$ K. The Nusselt number is defined as

$$
Nu = \frac{D \frac{\partial T - T_0}{\partial n}}{T_w - T_0},
$$

(15)
Fig. 4: Horizontal (left) and vertical (right) component of velocity, center-plane, $x/d = 15$

Fig. 5: Horizontal (left) and vertical (right) component of velocity, center-plane, $x/d = 20$
Fig. 6: Secondary flow in the plane $x = 20d$, SST model (left) and EARSM (right)

Fig. 7: Wall shear stress on impingement wall

where $n$ denotes wall normal direction. The result in terms of Nusselt number is shown in Fig. 11. All models over-predict the heat transfer in the stagnation region. This allows us a statement that even a sophisticated constitutive relation for Reynolds stress as well as for turbulent heat transfer hardly guarantees quantitatively acceptable results. Instead, the turbulent time scale prediction is of higher importance. We recall here a very good results of Merci, Dick [7] achieved with eddy diffusivity heat flux model, but with a non-linear $k-\epsilon$ model, where attention has been payed both to the constitutive relation as well as to the $\epsilon$-equation (i.e. time scale prediction).

Thus we can conclude that a more complex constitutive relations remain here limited by turbulent scale prediction, here the $\omega$-equation.

As for constitutive relations, the eddy diffusivity assumption leads to even qualitatively incorrect result with no drop of heat transfer at $r/D \approx 1.5$, analogous error as in wall friction. Other models predict the presence of the drop correctly, with GGDH even being in good agreement with experiment starting from this radial position.

A simple mean to improve the time scale prediction here could be the well known Yap correction, which (originally in a $k-\epsilon$ model) increases
Fig. 8: Velocity in the impinging jet at different radial positions

Fig. 9: Turbulence kinetic energy $k$ on impinging jet axis
Fig. 10: Reynolds shear stress in the impinging jet at different radial positions

Fig. 11: Nusselt number on the impingement wall
the source in the $\epsilon$-equation proportionally to $k$ and $y_\omega^3$ and thus diminishing the near wall over-prediction of $k$ in the impingement region. We added the analogous term $Y_c$ in the $\omega$-equation of EARSM (6) in the following form with the switching functions from [7]

$$\frac{D\omega}{Dt} = \frac{\omega}{k} P + \ldots + \frac{1}{C_\mu k} Y_{ce}$$

$$Y_{ce} = \begin{cases} 0, & S \leq 1.05\Omega, \\ 0.13(1 - f_{Ry}) \cdot \frac{k^2}{y^*} \max \left( \frac{0.4k^{1/2}}{y^*} - 1, 0 \right), & \text{otherwise} \end{cases}$$

$$S = \sqrt{2S_{ij}S_{ij}}, \quad \Omega = \sqrt{2\Omega_{ij}\Omega_{ij}}$$

$$f_{Ry} = \frac{1}{2} + \frac{1}{2} \sin \left( \frac{\pi}{2} \right) \cdot \min \left( \max \left\{ \frac{R_y}{500} - 3, -1 \right\}, 1 \right),$$

$$\epsilon = \beta^* k\omega, \quad \beta^* \equiv C_\mu = 0.09.$$
