FLOW MACRO-INSTABILITIES IN STIRRED TANKS:
STILL OPEN CHALLENGE

Milan Jahoda*, Pavel Hasal*, Ivan Fořt**
*Dept. Chem. Engng., Inst. of Chem. Technol., Prague, 166 28 Praha 6, Czech Republic,
**Dept. Proc. Engng., Czech Technical University, 166 07 Praha 6, Czech Republic
Corresponding author: Milan.Jahoda@vscht.cz, phone:+420220443223, fax: +420233337335

Keywords: stirred tank, flow macro-instability, mixing, turbulence, flow field

Abstract

An existence of low-frequency, large-scale variations of flow field in stirred tanks with a high-speed rotating impeller and radial baffles has been discovered a decade ago. The term macro-instability (MI) of the flow pattern has been assigned to this phenomenon. The macro-instability gives rise to significant pseudo-periodic variations of the flow field in the tank on a spatial scale comparable to the tank size and with the frequency substantially lower than the frequency of impeller rotation. Several kinds of the macro-instability have been identified in standard mixing tanks. The first kind of MI can be linked to recurrent fast liquid jets originating at the tank bottom and ascending upwards along the baffles and the tank wall. The second MI type is linked to the precessional motion of a swirling vortex (or vortices) around the impeller shaft. The MI of the third type originates due to flow pattern intermittency at certain impeller off-bottom clearances when the flow field switches between the single- and double-loop circulation patterns.

The MI presence in the flow field within the tank can be observed visually or by mechanical measuring devices monitoring the flow fluctuations. Most of experimental results, however, have been obtained by measuring the velocity fields using LDV or PIV methods. This paper focuses on a general review of experimental and theoretical studies of various aspects of the MI-related phenomena in order to demonstrate and summarize a depth and completeness of the macro-instability phenomenon understanding.

1 Introduction

Stirred tanks are widespread in most of process industries where they are used for a variety of purposes (blending of miscible liquids, dispersing one immiscible liquid into another, dispersing a gas into a liquid, suspending solid particles, enhancement of mass or heat transfer). Liquid flow in a mechanically stirred tank (see Fig.1 for standard mixing vessel configuration) is the subject of numerous experimental and numerical studies. Various aspects of the flow were studied, e.g., mean flow velocity, turbulent velocity field fluctuations (turbulence intensity), the rate of energy dissipation and time and space scales of

Fig. 1. Standard flat-bottomed cylindrical mixing tank with radial baffles. $T$… tank diameter; $H$… liquid filling height; $D$… impeller diameter; $C$ … impeller off-bottom clearance height; $b$ … baffle width. Standard dimension ratios: $H/T = 1$; $D/T = 1/3$; $b/T = 0.1$; $C/H = 0.2 – 0.5$. 


turbulence. Intensive research on flow field in stirred tanks has resulted to a wide knowledge of the stirring process enabling effective design of stirring equipment, e.g., from the point of view of power consumption or mixing time. Nowadays, complete velocity vector or turbulence intensity fields are available via measurement or numerical computations in many configurations of the stirring tank. We can state that the mean flow hydrodynamics in stirred tanks is quite well understood.

The flow structures in the stirred tanks are highly complex and comprise components vastly differing in their temporal and spatial scales. The spatial scales of particular flows in the tank extend from sizes comparable to tank dimensions to microscopic size of turbulent eddies that dissipate the kinetic energy of the flow. Liquid flow in a stirred tank hence can be viewed as a pseudo-stationary high-dimensional dynamical system consisting of a high number of hierarchically ordered unsteady flows – circulation loops, vortices and eddies. The temporal scales (life-times) of these flows span several decimal orders of magnitude.

The flow structure in a mixing tank depends on the vessel configuration and the impeller type. The Rushton turbine (RT) and the pitched blade turbines (PBT) - see Fig. 2 - belong to the mostly used impeller types. The hydrofoil impellers, e.g., Chemineer HE3 and Lightnin A310 - see Fig. 3 - have found wide range of applications during last decade due to their high mixing efficiency and low power input. Among the liquid flows encountered in the stirred tanks (see Fig. 4 for fundamental flow structures) one has attracted particular attention during the last decade. This flow occurs in a pseudo-periodic manner with the frequency markedly lower than the impeller frequency, and its size is comparable to the size of the tank. The flow was termed macro-instability (MI) of the flow pattern. The MI is characterized by space and time scales considerably exceeding the scales of common turbulent eddies in a stirred liquid. The size of the MI eddies is comparable to the size of the tank and its time scale is two or three orders of magnitude.

Fig. 2. 4-bladed Rushton turbine (radial impeller) and 4-bladed pitched blade turbine (axial impeller).

Fig. 3. The hydrofoil impellers Lightnin A310 and Chemineer HE3.

Fig. 4. a) Single-loop circulation pattern in a stirred tank with an axial impeller or a radial impeller at low off-bottom clearance. b) Double-loop circulation pattern in a stirred tank with a radial impeller.
magnitude longer than the period of one impeller revolution.

The MI existence has been indicated by temporal variations of the local gas hold-up [1], by pseudo-periodic variations of the heat transfer intensity at the tank wall [16], and by the variations of the flow-pattern structures [2,29,48]. Brůha et al. [4,5] have used a special mechanical device ("tornado-meter") to examine the flow structure variations and to determine the frequency of the MI occurrence. The image analysis methods [14,15] and visual observations [3] have been also used. The procedure most frequently used in the stirred tanks for detecting and quantifying the macro-instability consists in spectral analysis of local liquid velocity measured by means of the laser–Doppler velocimetry [10,11,13,19,23,40,42,47]. The presence of the MI in the flow pattern is evidenced by distinct peaks in the low-frequency part of the spectrum. Myers et al. [41] used a Digital Particle Image Velocimetry (DPIV) technique for an investigation of the flow field in stirred tank with axial-flow impeller. Transients among different flow patterns were observed occurring in a pseudo-periodic manner, with the period varying from 40 to 300 periods of the impeller revolutions. The DPIV technique for the MI observation was also used by Fan et al. [8,9].

2 Description of MI phenomena in mixing tanks

The macro-instability of the flow pattern has a strong impact on all mixing processes directly linked to fluid motions (mass- and heat-transport rates, local gas or solid hold-up values, homogenization rates, mixing time). The macro-instability has been identified as a mechanism responsible for liquid mixing outside the volume of the primary liquid circulation loop [3].

Three particular issues are addressed in the most of the studies: the characteristic MI frequency, the origin of the macro-instability existence, and the fraction of the kinetic energy of the flow captured by the macro-instability.

A linear dependence of the MI frequency, \( f_{MI} \), on the impeller speed, \( N \), has been identified in all experimental studies [4,5,10,11,19,23,40,42,47] – cf. Fig. 5, where experimental data of Hasal et al. [20-22] are replotted. A remarkable scatter of the MI frequency values reported in the literature, although they have been determined under similar or even the same tank and impeller configurations, has been emphasized by Galletti et al. [11] and by Nikiforaki et al. [42]. A part of the scatter can be assigned to inadequacies in spectral data evaluation and interpretation. The effects of the impeller type, of the tank configuration (impeller off-bottom clearance, relative impeller diameter), and of the liquid properties can explain another part of the frequency value.

![Fig. 5. Linear dependence of the macro-instability frequencies in mixing vessel with the Rushton turbine impeller at two off-bottom clearances: a) \( C/H = 0.35 \); b) \( C/H = 0.5 \), and various liquid viscosity and density.](image-url)
scatter. However, a significant part of the scatter is not completely explained yet. An existence of various kinds of the macro-instability is hypothesized as a reason of this unexplained variability. Galletti et al. [11,13] reported a coexistence of two characteristic MI frequencies in the power spectra of the local fluid velocity at various locations in the mixing tank with the Rushton turbine at the off-bottom clearance $C/T=0.5$. The higher frequency, of about $0.17 \, N$, occurred at lower impeller Reynolds number values (approximately for $Re_M < 20,000$) and the lower frequency, of about $0.02 \, N$, has been observed for $Re_M > 13400$. Within the interval $13400 < Re_M < 20,000$ both frequencies were present simultaneously. The intermittent switching of two circulation patterns in the tank [10,45,46] may explain this observation. The coexistence of two MI frequencies in the tank with the Rushton turbine has been also observed by Matsuda et al. [33-35].

Three kinds of the flow pattern instabilities have been identified so far in mixing tanks [10,42,45,47]. The first kind of the instability emerges primarily from the changes in the impeller off-bottom clearance when the flow pattern varies qualitatively, usually from a single- to a double-loop pattern [7,12,29,37-39,49]. The second type of the instability is associated with the changes of the impeller speed when the flow pattern in the tank with the pitched-blade turbine varies from mainly radial to mainly axial flow at certain critical $Re_M$ value. The third kind of flow instability [28,36,43] is generated by a "whirlpool" vortex (or vortices) slowly rotating around the impeller shaft with the frequency of about $0.02 \, N$. The simultaneous coexistence of several types of the macro-instability and their mutual interactions may give rise to a highly complex dynamical behavior of the liquid flow in the tank. Detailed comments on conditions of the occurrence and co-existence of two fundamental types of the macro-instability – the instability of the circulation loops (frequency of about $0.06 \, N – 0.2 \, N$) and the whirlpool vortex related instability (frequency about $0.02 \, N$) – were given by Kresta and Roussinova [28] and by Yianneskis [50]. The existence of the macro-instability of the flow pattern has not been confirmed only by experiments but also by liquid flow simulations based on applications of the CFD techniques [17,18,45].

The macro-instability not only affects various aspects of mixing processes related to the fluid motions, but it also generates dynamical forces affecting solid surfaces immersed in the stirred liquid. The MI-related forces vary with a frequency equal to the macro-instability frequency and produce slowly varying stresses within respective solid bodies that may, in certain cases, cause a failure of the tank, for example a baffle break-off. The tangential force affecting the radial baffles was measured by Kratěna et al. [27] in a standard mixing tank stirred with pitched blade turbines with four or six blades and with the Rushton turbine at several impeller off-bottom clearances over wide interval of the impeller Reynolds number value. Their measured data have been analyzed by Hasal et al. [20-22] who separated the MI-related component of the force from the data and determined the MI frequency and relative MI contribution to the total force. A numerical procedure based on a combination of the Proper Orthogonal Decomposition (POD) and spectral analysis has been used for this purpose. The same technique was adopted to the velocity data obtained by Montes et al. [19,23,40]. Deterministically chaotic features of the macro-instability have been uncovered and certain invariants of the MI chaotic attractors evaluated [19,23]. The chaotic nature of the macro-instability was also indicated in experimental data of other authors [31-33].

There are only few other attempts reported in the literature to separate and quantify the MI-related part of a data (time series) measured in a stirred tank. Letellier et al. [30] adopted a Hilbert transform based procedure for separating the low-dimensional deterministic part of an experimental time series. Kovacs et al. [26] used the Fourier transform for the same purpose. The kinetic energy of liquid captured by the macro-instability was evaluated in few papers. Roussinova et al. [47] employed a technique of data filtration in the time domain. By multiple application of a moving average
filter their measured velocity data were decomposed to the MI-related component, to the turbulent component and to the component corresponding to the impeller blade passage frequency. Typical contribution of the MI component to the total RMS velocity value varied from 7% to 30%. Nikiforaki et al. [42] performed a decomposition of the velocity data in the frequency domain. The relative contribution of the MI velocity component to the total RMS value varied from 10% to 25%. Matsuda et al. [31,32] evaluated rough estimates of a magnitude of the MI-related component of the fluid flow in a tank stirred with the Rushton turbine and pointed to strong dependence of the MI magnitude distribution within the tank on the impeller off-bottom clearance. The POD based procedure for data decomposition has been also used by Raju et al. [44].

3 Experimental studies of macro-instability phenomena in stirred tanks

For understanding the macro-instability phenomena in stirred tanks, the experimental investigation is essential as the numerical simulation methods (despite their rapid advancements) are still in their infancy. Various experimental methods have been used during the past 15 years. Here we present the principal experimental methods described in literature, which represent studies of the macro-instability phenomena at various research groups.

A visual observation of the flow pattern in the mixing tank represents the simplest method for the experimental MI studies. Winardi and Nagase [48] investigated unstable flow phenomena in their mixing tank with a three-blade commercial marine propeller using flow visualization. The tank was of approximately ellipsoidal shape (eccentricity 0.403 m : 0.396 m) with four wall baffles. Water has been used as a working liquid and the tank was filled to a height of 0.4 m. The impeller shaft was aligned and placed coincidently with the center-line of the tank. The impeller diameter was 0.16 m and its off-bottom clearance was 0.135 m. The impeller was rotated at a constant rotational speed of 180 rpm. Polystyrene particles (diameter of approximately 0.5 mm) were used for flow visualization. The flow observations were carried out by four observers watching simultaneously the circulation flow patterns near the tank wall within each quadrant between two adjacent baffles. A stable camera operated at a shutter speed of 1/8 second was used for observing the side view of the overall flow field in the tank.

Chapple and Kresta [7] used the tuft visualization in combination with a video recording to study the stability of the flow pattern generated by the aerofoil (Lightnin A310) impeller (cf. Fig. 3) and by the two- or four-bladed PBT (pitch angle 45°). The baffled cylindrical tank used in their work was of a diameter \( T = 0.24 \) m and was filled with water to a height equal to the inner tank diameter \( (H = T) \). The impeller diameter equaled \( D = T/2 \) or \( D = T/3 \) and the impeller off-bottom clearance varied from \( 1D \) to \( 0.6D \). The impeller rotational speed was fixed at 200 rpm in all experiments. For flow visualization, thirty-six 25 mm long tufts (made of an embroidery cotton) were placed in the tank. The tufts were suspended from thin wires spanned across the tank, above and below the impeller, and attached to the tank walls in the plane of two opposite baffles, and at 45° to this plane. The tuft locations were chosen to capture the major features of the top to bottom circulation patterns produced by the axial impellers. Tuft visualization data were collected for 20 min in each experiment. The data were analyzed continuously, one tuft at a time, starting from the same point in the recording for each tuft. The time series of flow directions were created manually by hitting a key on the computer keyboard each instant a new direction of flow has appeared. The time instants of the change in the flow direction (five directions of flow were defined: up, down, left, right, and orthogonal to the plane of observation) were recorded in the data file.

Jahoda et al. [25] observed the flow pattern near a baffle in a cylindrical tank of the inner
diameter $T = 0.29$ m by means of high-speed video cameras. Water at room temperature has been used as a working liquid and the liquid height was equal to the tank diameter. The impellers used were the six-bladed pitched blade ($45^\circ$) turbines (pumping down) of diameter either $D = T/3$ or $D = T/2$. The off-bottom clearance was $0.5D$. The impeller speed was varied in the range from 150 rpm to 500 rpm. The value of the Reynolds number for mixing was changed in the interval from 60,000 to 88,800. Two high-speed CCD cameras were used for a visual observation of the main flow and local liquid level swells in the baffle area. The first camera was placed horizontally and focussed on the inner edge of the baffle. The second camera was placed vertically and focused on the liquid surface (top view). Small red color spherical particles of diameter of 0.9 mm were added into the stirred liquid to make the flow pattern observation easier. The particles concentration was 0.2% by weight. For detailed observation of global flow structures, ten flow followers (plastic bodies of special shape) were also put to the liquid batch. The particles and the flow followers were of almost the same density as the stirred liquid. Using digital image analysis methods, the flow patterns near the baffle were extracted from the records of traces of the spherical particles and the flow followers.

In the turbulent region of mixing, the occurrence of the flow macro-instabilities in stirred tanks makes visible itself by a locally swollen liquid surface – the macro-swells (MS) – that obviously differ from the surface undulation originating due to the common (small scale) turbulent eddies. An electric conductivity measurement method was mainly used for the experimental investigation of the liquid surface macro-swells incidence. Brůha et al. [6] studied this phenomenon in a cylindrical baffled tank with diameter $T = 0.294$ m. Water at 20°C was used as the working liquid and the tank was filled to a height equal to the tank diameter. The vessel was provided with 3, 4 or 6 standard wall baffles. Four stirrers were used: three pitched blade ($45^\circ$) impellers pumping down (diameter $D = T/3$) with 3, 4 or 6 blades and the PBT of diameter $D = T/4$ with 4 blades. Three impeller off-bottom clearances were used: $C = 0.33T$, $0.4T$, and $0.5T$. The impeller speed was varied from 200 rpm to 600 rpm. The macro-swells occurrence frequency was monitored by means of a ring-shaped conductivity probe placed at a distance of 5 mm above the surface of the unstirred liquid within the area of macro-swells occurrence. When the water surface macro-swell contacted the probe the electric circuit became closed and a resulting electrical signal was recorded by a computer. The number of the macro-swells counted during the pre-selected time interval was monitored.

Jahoda et al. [25] have investigated dynamics of the local macro-swells of the liquid surface by means of a set of thin rod conductivity probes. The measuring probes were placed in the center of the area of local swells occurrence next to each baffle. The locations of the local macro-swells occurrence were detected visually. The ground sheet electrode was placed on the vessel bottom. The measuring probes were permanently immersed in the liquid and their output signal varied according to the level of their immersion. To prevent interference among the probes, the relay...
multiplexer board digitally controlled by PC-LabCard was used. The sampling frequency was about 450 Hz when only a single probe was used and 10 Hz when all (four) probes were simultaneously used in the experiments. After the finishing the data acquisition, the time series of actual liquid surface heights at all probes locations were stored and evaluated by a computer.

The macro-fluctuation of the liquid flow in the mixing tanks was also studied using mechanical detection devices. Brůha et al. [6,7] constructed an original device named "tornado-meter". The tornado-meter consists of a small circular target attached to a swinging rod-like arm connected with an electric resistance sensor. The device has detected the radial component of flow fluctuations and has been able to identify a shift between the single loop and the double loop flow patterns. Authors also performed visual observation of the swelling liquid surface. The experiments were carried out in a cylindrical vessel of diameter $T = 0.3$ m with four radial baffles. The tank was filled with the liquid up to the level height $H = T$. Water, aqueous sucrose solutions (27% and 46% by weight) were used in experiments. The PBT with 6 blades ($45^\circ$) of diameter $D = 0.1$ m ($D = T/3$) was used for stirring. Its off-bottom clearance was set to three levels: $C = 0.2T$, $0.35T$, and $0.5T$. The impeller speed was varied from 200 to 400 rpm. The liquid was kept at the ambient temperature. All experiments were performed in the turbulent region or in the early transition region of liquid flow.

Kratěna et al. [27] studied the mutual dynamic interactions between the impeller discharge flow of a radial impeller and a wall baffle in the tank in the turbulent regime of the fluid flow. The experiments were carried out in a cylindrical tank of the inner diameter of $T = 0.3$ m filled ($H = T$) with water or aqueous glycerol solutions (dynamic viscosity 3 mPa·s or 6 mPa·s). A standard Rushton turbine with six flat blades of diameter $D = T/3$ at the off-bottom clearance $C = T/3$ has been used in experiments. The impeller speed has been varied in the range from 187 to 350 rpm. The original method for measuring the peripheral (tangential) component of a dynamic pressure affecting the baffles was developed. One of the baffles was equipped with a small rectangular trailing target that was enabled to rotate along an auxiliary shaft parallel to the tank axis with a small eccentricity. The target was balanced by a couple of springs (see Fig. 6). The angular displacement of the target was directly proportional to the peripheral force affecting the target. By means of a photo-electronic sensor, the angular target displacement was converted to the voltage signal that was scanned and then stored and analyzed by the computer. The force was examined at eleven positions of the target along the baffle.

A majority of experimental results have been obtained by measuring the velocity fields using Laser Doppler Velocimetry (LDV) and Digital Particle Image Velocimetry (DPIV) methods. Kresta and Wood [29] measured radial, axial, and tangential velocity components using the LDV method with the 25 mW helium-neon laser operating in the forward scatter mode (at the sampling frequency of 2.5 kHz). The experiments were conducted in a fully baffled cylindrical mixing tank of diameter $T = 0.152$ m. The water height equaled the tank diameter. A four-bladed PBT ($45^\circ$) of diameter $D = T/2$ was located at the off-bottom clearance of $C = T/4$. All experiments were performed in the fully turbulent range of liquid flow at the impeller speed of 400 rpm. The liquid velocity was measured at an axial position 2 mm below the lower edge of the impeller blades, at the radial distance of 44 mm from the center of the tank.

Montes et al. [40] measured the axial and radial components of the liquid velocity in a vertical plane between two adjacent baffles in a mixing tank using the LDV technique. The rectangular grid of measuring points with 8 horizontal levels and with 11 vertical levels has been used. The experiments were carried out in the cylindrical, flat-bottomed tank with four radial baffles. The inner diameter of the tank was $T = 0.3$ m, the tank was filled with the working liquid to the height $H = T$. The six-bladed PBT ($45^\circ$) impeller of the diameter $D = T/3$ pumping downwards was used in the
experiments. The impeller off-bottom clearance was fixed at $C = 0.35T$. Water and aqueous glycerol solutions (85% and 91%) were used as working liquids in order to cover both transitional and turbulent mixing regimes. With water, the impeller speed was varied in the range from 450 to 600 rpm and with the glycerol solutions from 200 to 700 rpm. An Argon ion laser (3W) was used for liquid velocity measurement. The LDV was operated in a dual beam mode with the Bragg cell and the Doppler signal was analyzed using the PDA processor. The data recording time interval in each experiment necessary for two velocity component determination was about 25 min. At the data sampling rate of 330 samples per second about 500,000 experimental data points were obtained in a single experiment.

Bittorf and Kresta [3] studied an active volume of mean liquid circulation in the mixing tanks with axial impellers by the LDV method. Two fully baffled tanks of diameters $T = 0.24$ m and 0.14 m were used. A free liquid (water) surface was used to eliminate any effects due to the presence of the tank lid. The 0.24 m tank was filled to the liquid height equivalent to tank diameter. The 0.14 m tank was filled with the liquid to a height of $1.21T$ to minimize the air entrainment. Instantaneous axial velocity was measured using an Argon ion laser (300 mW) in front of the baffle at distance of 5 mm from the baffle. The rectangular grid of measuring points with 35 horizontal levels and with 6 vertical levels was used. Total measuring time interval ranged from 14 to 45 s with mean measuring time of 20 s. The total sample number was 8000 points. Three axial impellers were used for stirring: a four-bladed (45°) PBT, the Chemineer HE-3 impeller, and the Lightnin A-310 impeller. The impeller diameters ranged from $D = 0.19T$ to $D = 0.58T$. Three to five impeller off-bottom clearances and two rotational speeds were examined with each impeller. Altogether 66 experimental configurations were explored. Water and Bayol-35 were used as the working liquids. The impeller Reynolds number ranged from $1.35 \cdot 10^4$ to $1.96 \cdot 10^7$ with the majority of the experiments done at a Reynolds number above $2 \cdot 10^5$.

Roussinova et al. [45] examined the effects of the tank scale on MI phenomenon using data from a tank with diameter $T = 1.22$ m. The radial baffles were equally spaced along the tank perimeter at a small distance from the wall. The liquid level in the tank was kept equal to the tank diameter. A four-bladed PBT (45°) impeller of diameter $D = T/2$ at three off-bottom clearances ($C = 0.165T, 0.25T$, and $0.33T$) was used in experiments. Three working liquids were used: water, Bayol-35 and a solution of triethylene glycol in water. The measuring region within the tank was located upstream of the baffle and coincides with a region of the developing three-dimensional wall jet. 60 experimental points were monitored. The velocity time series were collected using a two-color LDV system. The total number of collected samples per measuring point varied from 15,000 to 20,000 giving the total recording time from 2 to 10 minutes.

Nikiforaki et al. [42] investigated the mean flow and turbulence fields in a fully baffled tank stirred by a Rushton turbine and a PBT using the LDV technique. The experiments were performed in a cylindrical tank of diameter $T = 0.294$ m, filled with water to a height equal to the tank diameter and equipped with four equally spaced baffles. Two impellers were used: a six-blade RT of the diameter $D = T/3$ and the six-blade PBT (60°) of the same diameter. The experiments were done at the impeller off-bottom clearances $C = 0.15T$, $0.33T$, and $0.5T$ for the Rushton turbine and $C = 0.33T$ for the PBT. The impeller speed was fixed at 250 rpm for RT and at 350 rpm for PBT. For Rushton turbine located at $C = T/2$ the measuring point was located at axial position $z = 0.88T$ above the tank bottom and at radial distance $r = 0.11T$ from the tank axis. For PBT the sampling point was located at $z = 0.22T$ and $r = 0.13T$. The locations of the sampling points were chosen by visual inspections of the flow field. For each experimental point large number of data points (varying from 50,000 to 1,000,000) was obtained, total recording times were varied from 10 to 800 s. The sampling rate was varied from 0.5 to 2 kHz (mostly around 1 kHz).
Galletti et al. [12] carried out an extensive set of the LDV measurements in order to elucidate the origin of MI in stirred tanks. Measurements were taken in a cylindrical fully baffled stirred tank of the inner diameter $T = 0.29$ m filled with water to a height equal to the tank diameter and covered with a lid. The stirring was provided by either a Rushton turbine of diameter $D = 0.34T$ or four-bladed PBT of diameter $D = 0.47T$. The off-bottom clearance of the RT was varied between $C = 0.15T$ and 0.2$T$, whereas the PBT was fixed at $C = 0.25T$. The impeller speed was varied between 70 and 400 rpm. The measuring system consisted of a four-channel laser. The beam from a 3W Argon-ion laser was separated into green and purple lines. Both beams were transmitted by means of optical fibers to two separate probes. In case of experiments with the RT, the first probe was positioned at $r = 0.22T$ and $z = 0.13T$ to measure the radial velocity component; the second probe was positioned at $r = 0.1T$ and $z = 0.9T$ to measure the axial-tangential velocity component. In a case of the experiments with PBT, the first probe was used to measure the horizontal component of the liquid velocity in a vertical plane located upstream of a baffle and at three horizontal planes ($z = 0.05T, 0.6T, 0.93T$). The second probe measured the axial velocity component at another location in the vessel. The total measuring time was 900 s and the data sampling rate was 150-1200 Hz.

Myers et al. [41] used a digital particle image velocimetry (DPIV) technique for an investigation of the flow field in a stirred tank. The diameter of their cylindrical tank was $T = 0.292$ m. The tank was equipped with four baffles evenly spaced along the perimeter of the tank. The baffle width equaled one-twelfth of the tank diameter. The baffles were offset from the tank wall by a distance equal to one-sixth of the baffle width. Water was used as the working liquid and its level height was equal to the tank diameter. Two impellers were used: a four-bladed PBT (45°) of diameter $D = 0.35T$ at the off-bottom clearances $C = 0.46T$ and 0.33$T$ and the Chemineer HE-3 impeller of diameter $D = 0.39T$ at the off-bottom clearance $C = 0.33T$. The impeller speed was varied from 36 to 60 rpm. The DPIV method with the time delay of 0.033 s between the consecutive images was used. Series of either 64 or 1024 image pairs were taken. The successive image pairs were separated by a time period of approximately 11 seconds when 64 image pairs were taken and by the period of 1.3 second for the 1024 image pairs. These values yielded the total recording time of 700 and 1330 seconds. Time series analysis of recorded data was performed only with the data sets obtained at higher sampling rate (1.3 s between successive image pairs).

Fan et al. [8,9] used a DPIV technique to measure the near-instantaneous flow fields in a fully baffled tank, equipped with a four-bladed Rushton turbine. The tank diameter value was $T = 0.28$ m and the tank was filled with water or an aqueous glycerol solution (9–100% of glycerol by volume) up to the height $H = T$. The impeller diameter was $D = T/2$ and its off-bottom clearance was $C = T/2$. The impeller speed was fixed at 30 rpm. The measurement area was located in a vertical plane determined by two opposite baffles or in the mid-plane

![Fig. 7. Autocorrelation functions of tangential force affecting the baffles in a stirred tank exhibiting: a) strong low-frequency periodicity, b) fast exponential decay (suppressed periodicity).](image-url)
between two adjacent baffles, respectively. In the course of the experiments 1024 successive DPIV measurements were typically taken using a time delay of about 0.625 ms, resulting to the total measurement period of approximately 11 min.

Guillard *et al.* [14,15] used a fluorescence technique to study the turbulent mixing in a tank equipped with two standard Rushton turbine. The fully baffled tank of an inner diameter $T = 0.3$ m was filled with water up to the height $H = 2.15T$. The diameter of the Rushton turbines was $D = T/3$. The lower impeller was located at the off-bottom clearance $C = 0.55T$ and the spacing between both impellers was $0.7T$. The impeller speed was set to 294 or 396 rpm. Planar Laser Induced Fluorescence method was used to determine the mixing rate. Experiments were carried out with the tracer injection at the point situated at $r = 0.4T$ and $z = 1.85T$ (the edge of a baffle). Three measurement planes located at the distances of $0.014T$, $0.116T$ and $0.336T$ from the injection point were chosen. The injection system delivered an aqueous solution of Rhodamine B at constant flow rate of $0.8$ ml·s$^{-1}$. The fluorescence emitted by the excited Rhodamine B was captured by a CCD camera. The grabbing rate was set to 1.67 images per second in all experiments.

### 4 Analysis of experimental MI-related data

The experimental data resulting from experimental studies of the macro-instability related phenomena in mixing tanks predominantly take a form of uni- or multi-variate time series of the liquid velocity components with either equidistant or non-equidistant data spacing. Another time series are obtained by measuring the force acting on the baffles or special mechanical devices or by recording the torque acting on the impeller shaft or the tank wall. Sequences of image data (series of 2D data matrices) result from an observation of the flow structures by means of PIV or DPIV methods when a selected region of the flow field is continuously recorded by a videocamera. Each kind of the data requires adequate methods of analysis. The methods reported most frequently in the literature are briefly summarized and sorted in following paragraphs.

#### 3.1 Statistical analysis

The statistical analysis of time series typically represents an introductory step in an evaluation of any experimental MI-related data. The evaluation of mean values, variances, standard deviations or RMS values of the observed quantity and their spatial distribution within the mixing tank gives first quantitative insight to the significance of the flow variations due to the presence of macro-instability with respect to the mean liquid flow. The mean value of the square of the velocity fluctuations gives the kinetic energy of the fluctuation component of the liquid flow and can thus be used as a quantitative measure of a contribution of the fluctuating flow component to the total flow kinetic energy [10,11,13,42,47].

![Fig. 8. Power spectral densities of tangential force affecting the baffles in a stirred tank with the pitched blade turbine (a) and with the Rushton turbine impeller (b).](image)

The evaluation of probability distribution functions (histograms or cumulative histograms) and of their higher statistical moments (skewness, kurtosis) represents further data analysis option that may yield additional information about nature of the flow field in the
mixing tank, e.g. about anisotropy of the flow field and of corresponding MI fluctuations [44].

Temporal and spatial links between liquid velocity fluctuations are commonly characterized by means of the correlation functions. The autocorrelation functions, when evaluated on basis of the experimental data containing the MI-related fluctuations exhibit more or less pronounced periodicity and (in certain cases) can be used to determine the MI frequency. Fig. 7 shows an example of autocorrelation functions evaluated both for data with the pronounced MI component (Fig. 7a) and for the data of strongly stochastic nature (Fig. 7b) where the MI component is suppressed [20-22].

3.2 Spectral and wavelet analysis

The evaluation of power spectral density of the analyzed experimental data represents basic tool in MI phenomena analysis. The power spectra are used for detection of presence of the MI in the experimental data, that exhibits itself by distinct peak (or a pair of peaks) in the low-frequency region of the spectrum. Fig. 8 shows power spectra with a single (Fig. 8a) MI-related peak and with two MI-related peaks (Fig. 8b). The power spectra also serves for determination of the MI frequency (frequencies). Very good spectral resolution in the spectra is therefore required. The spectra are typically evaluated using the Fast Fourier Transform (FFT) based algorithms. The values of parameters of the FFT based algorithms have to be carefully chosen to avoid misleading or even erroneous results and conclusions [24]. The sampling period of the data, total number of samples and their segmentation are the basic parameters for the evaluation of the spectra using the FFT algorithm. The FFT algorithm requires evenly sampled data points. The experimental techniques used for the data gathering (LDV, PIV), however, often yield data with the unevenly spaced samples. Then the algorithms based on an application of the Lomb's method [45,46] can be applied or numerical data resampling algorithm can be used to convert raw experimental data to the evenly sampled data sets [10,11,13].

The power spectra provide us only with information on presence or non-presence of certain frequency components in the analyzed data. The information on their temporal occurrence within the data collection interval is missing in the spectra. The flow structure in a stirred tank, however, quite often exhibits remarkable intermittency due to, for example, aperiodic switching between the single- and double-loop circulation pattern [7,11,29,34,37-39,42,46]. In such cases the wavelet transform (WT) based techniques can be used. The wavelet transform yields simultaneous time and frequency resolution of the analyzed data that is suitable for analysis of certain MI phenomena occurring under specific operating conditions of the mixing tanks [7,10,37].

3.3 Data decomposition techniques

The macro-instability phenomena are typically superimposed to the turbulent (stochastic) components of the fluid flow in the tank in raw experimental data. A kind of the decomposition has to be applied to the data when the contribution of the macro-instability component to the total flow, expressed by a suitable quantity (e.g., by kinetic energy of the flow) is sought for. Several decomposition techniques have been used for mixing data analysis by a number of authors. In macro-instability phenomena analysis the task of the data decomposition consists in separation of a low-frequency MI-related (deterministic) component from the turbulent (stochastic) component of the flow. A liquid velocity component is typically measured, and a fraction of the flow kinetic energy captured by the macro-instability is evaluated.

Letellier et al. [30] have adopted a Hilbert transform based procedure to perform the decomposition of an experimental time series from the mixing vessel. Kovacs et al. [26] used the Fourier transform for the same purpose.

Roussinova et al. [47] employed a data filtration in the time domain. By multiple application of a moving average filter the data
were decomposed to the MI-related component, to the turbulent component and to the component corresponding to the impeller blade passage frequency. Typical contribution of the MI component to the total RMS velocity value varied from 7% to 30%.

Nikiforaki et al. [42] performed a decomposition of the velocity data in the frequency domain. They detected respective peaks in the power spectra of the data and then applied an inverse Fourier transform to the data adjusted by subtracting the respective peak(s). The relative contribution of the MI velocity component to the total RMS velocity value varied from 10% to 25%.

Hasal et al. [19-23] have employed another technique consisting in a combination of spectral analysis with the Proper Orthogonal Decomposition. In this technique the experimental time series is centered by subtracting the mean data value. Then the centered data is transformed to the so-called trajectory matrix by repeated application of a n-window [23]. The length of the n-window defines frequency resolution of the POD analysis. The trajectory matrix is then used for evaluation of the autocovariance matrix of the centered data. The eigenvectors of the trajectory matrix are used to evaluate the eigenmodes \( a_k(t) \) of the centered data series. The measured data set is in this way decomposed to a set of \( n \) eigenmodes (see example in Fig. 9 where the eigenvalues and eigenmodes are presented together with the original data). Each eigenmode represents a time series and the eigenmodes may be used to reconstruct the original data. A value of the \( k \)-th eigenvalue, \( \alpha_k \), expresses relative contribution of the respective eigenmode to the variability of the analyzed data. When velocity data are analyzed the eigenvalue \( \alpha_k \) is proportional to a fraction of the kinetic energy of the flow variations captured by the \( k \)-th eigenmode. The POD eigenvalues can be used to estimate a relative magnitude of the macro-instability contribution to the measured data variability. The eigenmodes contributing to the macro-instability are chosen using their power spectra and the spectrum of the analyzed data. This way of resolving the frequency components of the data has proved as quite efficient and unambiguous method [19-23]. A decomposition of the data from a mixing tank to the eigenmodes has been also applied by Raju et al. [44] who analyzed data obtained by the stereoscopic particle image velocimetry.

The POD technique is an efficient tool for experimental data analysis but can also be used for a construction of low dimensional mathematical models for MI flows simulations.

### 3.4 Nonlinear MI data analysis

Several authors noticed analyzing their experimental data deterministically chaotic features of the MI-related phenomena [19,23,30-33]. Their results suggest that the macro-instability in stirred tanks is generated by a low dimensional dynamical system with chaotic dynamics. A few attempts have been done to evaluate chaotic invariants of the chaotic attractors reconstructed from the experimental data [19,23,31-33]. The attractors have been reconstructed mostly by the method of delays with the delay value determined from the autocorrelation function or function of mutual information of original data [31-33]. Hasal et al. [19,23] made use of the POD eigenmodes for the same purpose. An example of the attractor constructed in this way is shown in Fig. 10. The embedding dimension of the
phase space for an attractor reconstruction has been estimated by the false-nearest-neighbor method [19]. The embedding dimension value ranging from 3–5 has been found as sufficient for the attractor reconstruction. The macro-instability can therefore be really entitled as the low-dimensional phenomenon. Low dimensionality of the MI phenomena has been also confirmed by low values (between 2 and 5) of the correlation dimension of their attractors [19,23,31,32]. A positive value of maximum Lyapunov exponent of the attractors has been observed by some authors [19,32]. This fact supports a hypothesis on the chaotic nature of the macro-instability. More intense research of non-linear dynamics of the MI flows in mixing tanks is highly desirable as it may bring completely new insights to inherent MI properties and its dynamical behavior.

5 CFD modeling of MI phenomena

The presence of low-frequency flow fluctuations in mixing tanks related to the macro-instability has serious implications for computational flow modeling based on applications of CFD (Computational Fluid Dynamics) methods. Till now, only a few papers describing CFD modeling of the flow macro-instabilities in stirred tanks [17,18,45] have been published (time-consuming computations may be a reason of this fact). The Large Eddy Simulation (LES) method has been predominantly used in the CFD simulations performed to date.

Generally, in a turbulent regime of fluid flow in stirred tanks, the fluid moves in a form of fluctuating eddies, which create a random (chaotic) behavior of the velocity field. The size of the largest eddies is typically comparable to the characteristic length of the mean flow (the circulation loops etc.). The eddies of the smallest scale are responsible for the dissipation of the fluid kinetic energy. Theoretically it is possible (the Navier-Stokes equations can explicitly model fluid flow also in the turbulent regime) directly resolve the whole spectrum of turbulent scales in the flow using the approach known as the direct numerical simulation (DNS). For an application of this method, however, it is essential to use the cells in the computational grid of the same size as the size of the smallest eddies in the flow. Nowadays, the computing power of computers available for CFD studies is not sufficient to simulate the flow using DNS within reasonable time interval of several days and other approaches to DNS are sought. Two alternative methods can be employed to render the Navier-Stokes equations tractable in such a way that the small-scale turbulent fluctuations do not have to be directly simulated: a Reynolds-averaging (or ensemble-averaging) and filtering based on the Large Eddy Simulation (LES) method.

The Reynolds-averaged Navier-Stokes (RANS) equations govern transport of the averaged flow quantities using the whole range of the spatial scales of turbulence being modeled. Several well-known turbulence
models are commonly applied: the “$k$-$\varepsilon$” model and its modifications, the “$k$-$\omega$” model and its variants, and the RSM model (Reynolds stress model). But the RANS approach is not suitable for the macro-instability simulations.

The LES based methods provide an alternative approach to the modeling of the flows involving the macro-instability where large eddies are explicitly computed (resolved) in a time-dependent simulation using the “filtered” Navier-Stokes equations. Their solution requires less computing time than the DNS because it does not make use of fine computing grid as it models only the eddies greater than the single cell of the grid. The rationale behind the LES methods is that by the modeling less of turbulence (and resolving more), the error introduced by the turbulence modeling can be reduced. In LES, the large eddies are resolved directly, while the small eddies are modeled. Large eddy simulations thus fall between the DNS and the RANS methods in terms of the fraction of the resolved spatial scales.

The core of the LES methods can be summarized as follows: momentum, mass, energy and other passive scalars are transported in the fluid flows mostly by large eddies. The large eddies are markedly system-dependent. They are dictated by system geometry and by the boundary conditions of the flow. The small eddies are less dependent on the system geometry, tend to be more isotropic, and their properties are, consequently, more universal, i.e., independent of the modeled system. The chance of finding a universal turbulence model is much higher for the small turbulent eddies.

Resolving only the large eddies allows one to use much coarser mesh and larger time-step sizes in LES than in DNS. However, LES still require substantially finer computational meshes to be used than those typically used for the RANS calculations [51].

Roussinova et al. [45] have used the software package FLUENT v5.4 for their LES simulations. The impeller motion was modeled using the sliding mesh model. The computational model included full 3D grid with a total of 500,000 unstructured hexahedral cells. The simulation model was started from the zero-velocity field and 78 revolutions of the impeller (23.40 s of flow time) were simulated with the implicit time step of 0.01 s. Comparing to the LDV measurements the large eddy simulations were able to predict not only the mean velocity field but also the transient macro-instability fluctuations.

Hartmann et al. [18] have investigated motions of the precessing vortex in a mixing tank by means of the LES methodology. The vortex motion in the tank represents a transient phenomenon, which cannot be captured by the quasi-state simulation. Authors avoided the interface models commonly applied for the calculation of the stirred tank flows in the commercial codes, since it is unclear how significantly is the resolution of the precessing vortex motion affected by the location of the interface between the rotating and the stationary meshes and by the kind of the interface model itself. The LES methodology was embedded within the lattice-Boltzmann scheme and the Smagorinsky subgrid-scale model was used for discretization of the Navier-Stokes equations. The simulations were started from either a zero-velocity field, or from the flow field resulting from previous simulation. After roughly 20-30 impeller revolutions a quasi steady state has been reached. The main conclusion of Hartmann et al. [18] was that a grid of size of $180^3$ lattice nodes is necessary and sufficient for resolving the vortex motion at the Reynolds number values (20,000 and 30,000) investigated.

6 Conclusions

A considerable experimental, theoretical and modeling efforts have been devoted to the macro-instability phenomenon in stirred tanks since its discovery in the last decade of the 20\textsuperscript{th} century. The number of journal papers (mostly listed in the References section of this paper) and the number of the conference contributions dealing with various facets of the macro-instability related flows make a clear evidence
FLOW MACRO-INSTABILITIES IN STIRRED TANKS

of the importance and scientific attractiveness of the MI-related processes in stirred tanks. A considerable knowledge on the macro-instability occurrence frequency, on the MI-variations with the tank/impeller configuration and on MI contribution to the kinetic energy of the liquid or to its force effects on solid bodies has been gathered. Possible origins of the macro-instability occurrence and several mechanisms of its spatio-temporal development have been hypothesized and several distinct MI kinds have been identified accordingly. The macro-instability existence in the flow field in mixing tanks has been confirmed by several CFD based simulation studies.

Nevertheless, despite the advancements achieved heretofore in the MI studies there is still a number of unresolved problems concerning various MI aspects and the macro-instability of the flow pattern in mixing tanks still persists as an open challenge for members of the scientific community engaged in the research of mixing processes.

Acknowledgement: This project has been supported by the Czech Science Foundation (Grant: 104/05/2500) and by the Czech Ministry of Education (Grant: MSM 6046137306).

7 Notation

- \( b \) baffle width [m]
- \( C \) impeller off-bottom clearance [m]
- \( D \) impeller diameter [m]
- \( f_{MI} \) MI occurrence frequency [Hz]
- \( H \) liquid filling height [m]
- \( N \) frequency of impeller rotation [Hz]
- \( \text{Re}_i \) impeller Reynolds number [-]
- \( T \) tank diameter [m]
- \( r \) radial coordinate of the tank [m]
- \( z \) axial coordinate of the tank [m]

8 Abbreviations

- CFD Computational Fluid Dynamics
- DNS Direct Numerical Simulation
- DPIV Digital Particle Image Velocimetry
- FFT Fast Fourier Transform
- LDV Laser Doppler Velocimetry
- LES Large Eddy Simulation
- MI Macro-Instability
- PBT Pitched Blade turbine
- PIV Particle Image Velocimetry
- PDA Particle Dynamics Analyzer
- POD Proper Orthogonal Decomposition
- PSD Power Spectral Density
- RANS Reynolds Averaged Navier-Stokes
- RMS Root Mean Square value
- RSM Reynolds Stress Model
- RT Rushton Turbine
- WT Wavelet Transform

9 References


FLOW MACRO-INSTABILITIES IN STIRRED TANKS


